Advice for Instruction | 1. Exploring problem-solving strategies | Prepare instruction

Prepare instruction

Goals and objectives

In this topic, students experience collaboration as a strategy to solve problems. They share problem-solving strategies as they explore problems that have one or more solutions. In the topic Exploring problem-solving strategies, students will

- Introduce themselves and learn about their teacher, classmates, and the course;
- Learn and practice norms and routines to help them collaborate and learn;
- Explore, apply, and share problem-solving strategies, and reflect on solutions; and
- Work with important algebra ideas, including variables, patterns, and solutions.

The Staying Sharp problems in this topic are organized around several key ideas:

- Problems 1 and 2 (Practicing algebra skills and concepts): Solving shape equations and patterns
- Problems 3 and 4 (Preparing for upcoming lessons): Proportional reasoning and informal solving
- Problems 5 and 6 (Reviewing pre-algebra ideas): Operations with decimals and measurement

The focus skill for this unit, signed number operations, is introduced in Lesson 1.5 “Problem solving with patterns” as students use patterns in addition and subtraction of signed numbers.

Topic overview

Lesson 1.1: Students are introduced to the teacher, to each other, and to some important routines. Students work in pairs to solve shape equation puzzles and to consider the idea of variables.

Lesson 1.2: Students consider some ideas related to problem solving and work with the teacher to co-construct criteria for effective group work. They work with their partner on the Bike and Skateboard Problem.

Lesson 1.3: Students establish criteria for effective presentations and present solutions to the Bike and Skateboard Problem.
Resources

LESSON RESOURCES

- Computer with projection device and Internet connection
- Graphing calculators
- Large chart paper
- Masking tape
- Chart markers
- Sticky notes
- Whiteboards, dry erase markers, and erasers
- Pocket folders, one for each pair of students

Print resources

- [Unit 1 Topic 1 Student Activity Book](updated 4/29/14)
- [Unit 1 Topic 1 Student Activity Book answer key](updated 4/29/14)
Deliver instruction (Lesson 1.1)

Lesson materials

- Lesson 1.1 “Collaborating around algebra”
- Student Activity Book

Lesson preview

<table>
<thead>
<tr>
<th>Suggested time</th>
<th>Activity</th>
<th>Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 min</td>
<td>Welcome</td>
<td>Learn about the course, and about the procedures and routines that will be used in this class</td>
</tr>
<tr>
<td>10 min</td>
<td>Opener</td>
<td>Apply logic and reasoning to solve a mathematical puzzle</td>
</tr>
<tr>
<td>25 min</td>
<td>Core activity</td>
<td>Practice mathematical reasoning and communication when working with a partner</td>
</tr>
<tr>
<td>15 min</td>
<td>Consolidation activity</td>
<td>Learn how to use variables to represent unknowns</td>
</tr>
<tr>
<td>20 min</td>
<td>Wrap up and introduce homework</td>
<td>Reflect on the day’s lesson and understand tonight’s homework assignment</td>
</tr>
</tbody>
</table>

Lesson activities

WELCOME (10 minutes)

Students learn about the course, and about classroom expectations, procedures, and routines.

By outlining the goals of this course, you can “make learning explicit” for students. This is a major instructional theme of the course. While all learners benefit from this principle, it is especially important for struggling learners.

- *Intensified Algebra I* incorporates a significant number of daily routines. Routines and structure are important for struggling learners, and can help teachers facilitate instruction. This unit builds in opportunities for students to “co-construct” class routines, policies, and expectations. Therefore, on the first day you should communicate only the most necessary
expectations and policies.

Online page 1

- Use this page to outline the goals for this course.
- Discuss essential course policies and expectations (e.g., handling supplies and the homework and grading policies) and distribute any first-day handouts you have prepared, such as a course syllabus.
- Establish a routine for how class will begin each day. An expectation should be that every student is seated and working on the Opener when the bell rings to start class.
- **Classroom strategy.** Plan your seating arrangement for students in advance of the first day of class. Many of the learning activities in this program are based on students working in pairs. It is recommended that you assign partners (rather than allow students to choose their own partners) and arrange the desks in pairs.

**OPENER (10 minutes)**

Students use logic and reasoning to solve a shape equation puzzle — a puzzle with a set of equations in which shapes (like circles and squares) are placeholders for numbers.

Shape equation puzzles will be used throughout the course. They will be used in the unit on systems of equations, for example, as a way to develop the substitution method for solving systems. This activity is one of many intended to access students’ prior knowledge (e.g., with mathematical structures, number sense, logic, and reasoning) in a way that connects to algebraic thinking. One of the goals of this unit is to “build bridges” from middle school mathematics to algebra.

Online page 2

- Remind students that every lesson will begin with a 10-minute Opener, and it is therefore important that they begin working each day on these activities as soon as they are seated. Read through today’s Opener as a class. Point out the pencil icon with the numbers 1-2 on the top of the page, right side. This icon will be routinely used throughout the course to indicate to students that they will be working on problems in their Student Activity Book.
- Then, give students a few minutes to work individually on the opener. **[SAB, questions 1-2]** Encourage students to use logic and reasoning to solve the puzzle. The goal of this activity is for students to apply logic, informal reasoning, and algebraic thinking to solve these puzzles. Do not expect students to apply formal algebraic techniques to solve the puzzle. Also, do not overlay formal algebraic solutions to the puzzle in the debriefing of the activity, as that is not the intention of the activity.
- When you debrief the activity, emphasize that shape equation puzzles involve algebraic thinking — the type of thinking that is important in algebra.

Online page 3

- Preview the activities and learning goals for the day’s lesson.

**CORE ACTIVITY (25 minutes)**

Partners interview each other and then make introductions to the class. Students then work with their partners to solve more shape equation puzzles. Emphasis is placed on mathematical reasoning.
and communication.

**Online page 4**

Students interview their partners and then introduce their partners to the class.

- Give students one minute to interview their partner and record a few pieces of information based on the questions on this page. After one minute, call time and ask students to switch roles.
- Allow each student to introduce his or her partner to the class. Have each student pair stand up when it is their turn for introductions. You may want limit each introduction to 30 seconds.

**Online pages 5-6**

Students work with their partner to solve more shape equation puzzles.

- Page 5: Have partners work together to solve more shape equation puzzles. As a class, read the directions for the activity. Emphasize the importance of collaboration and communication in this activity. [SAB, questions 1-4] As students work on the puzzles, circulate around the room, asking questions and checking for understanding. Be sure to ask questions about how students approach solving the problem:
  - What have you tried? What steps have you taken?
  - What hasn’t worked?
  - Do you have a strategy that you think might work?
  - What information can you get from this equation that might help you?
  - Would it help to ... (use guess-and-check? start with a different equation? try substituting shapes?)

Pull the class together for a brief discussion after each pair of students has had an opportunity to work through the first puzzle. Students do not need to complete all four puzzles. Ideally, students will complete puzzles 1-3. Puzzles 1 and 2 can be solved with simple number substitutions; puzzles 3 and 4 can be solved by guess-and-check or by substituting shape expressions.

- **Classroom strategy.** If many students are struggling on a particular activity, a good strategy is to bring the class together for a brief discussion after they have had some time to work on the problem. You can work as a class to move students’ understanding forward by making use of student ideas and strategies, particularly those students who have demonstrated an “on-track” start to a problem. The goal is to provide scaffolds for learning without stripping away the cognitive demand.

- Page 6: Use this page to debrief the activity. Call on pairs of students to report their answers. Students should explain their solution and the process that they used to solve the puzzle. Here are some possible questions to ask:
  - How confident are you that your answer is correct on a scale of 1 to 10, with 10 being complete certainty?
  - How many other groups agree with that answer?
  - Has each clue’s condition been satisfied?

- **Classroom strategy.** Through questioning as modeled above, you can build and reinforce course expectations related to (a) the source of authority for right and wrong answers; (b) the development of a community of learners; and (c) the ways students can check their own work. A goal of the course is to develop students’ abilities to work on problems as...
mathematicians do. In developing these dispositions and work habits, “rightness” or “wrongness” is derived from the learning community, as a result of people working on problems and then presenting and discussing approaches and solutions. Cultivating these mindsets and habits often requires time and hard work, as these ideas may conflict with many students’ prior experiences and beliefs that the teacher is the only source for determining whether an answer is “right” or “wrong.”

CONSOLIDATION ACTIVITY (15 minutes)

Students are introduced to the idea of a variable as a placeholder for an unknown. Students are also introduced to various ways that multiplication is represented in algebra including the dot, ·, and parentheses.

Online page 7

Introduce the idea of a variable as a placeholder for an unknown.

• Play through the animation on this page to introduce the concept of a variable and to show students an example of how letters can be used as variables to replace shapes in a shape equation puzzle.

Online page 8

Students rewrite shape equations using letters as variables.

• Read the directions for the activity as a class and allow students time to work on the activity.
  [SAB, questions 1-2]
• Debrief the activity quickly. You may want to have several students place an answer to one of the questions on whiteboards and then post the whiteboards to share with the class.
• Ask several students how they represented the multiplication that occurs in the third clue of puzzle 2. Point out to students that there are different ways to do this, but they all mean the same thing.

WRAP UP AND INTRODUCE HOMEWORK (20 minutes)

Students learn the routine that will be used to wrap up and introduce homework at the end of each class; review today’s learning; and preview tonight’s homework.

A program goal is to build students’ abilities and confidence to complete homework activities independently. Therefore, the homework activities are designed to be meaningful and doable. *Staying Sharps* are meant to be part of the homework assignment for each lesson. However, you may choose to use them in different ways. For example, you may allow students who complete a class activity early to start them.

You are not expected to pre-teach or re-teach the concepts and skills embedded in the *Staying Sharp* assignments. The purpose is to gather formative data about students’ math strengths and weaknesses with respect to pre-algebra topics. This data can be used as you proceed in the course as a tool for formulating action plans for your teaching and intervention strategies for individual students and groups of students.

Online page 9
• Present the routine that will be used to introduce homework at the end of class each day.

Introduce tonight’s homework:

• **Homework 1.1**
  The homework provides additional practice with shape equations, and asks students to write about the routine established for starting class each day. You might add that following the routine when they come into class tomorrow is a part of the homework assignment.

• **Staying Sharp 1.1**
  Describe the main goal of *Staying Sharps*: To provide opportunities for on-going practice of previously-learned concepts and skills from both middle school mathematics topics, and, as the year progresses, topics from this algebra course.
  Another goal of the *Staying Sharp* activities is to provide information to the student and the teacher about areas of strength and weakness for particular concepts and skills. While students may not be able to do every problem in every *Staying Sharp* activity, they should make a good effort and take responsibility for identifying concepts and skills that they still need to work on.
  The main concepts and skills in these problems are:
  1. Solving a system of two shape equations (with addition and subtraction)
  2. Continuing and describing a pattern (of regular polygons)
  3. Identifying a fraction from a visual representation
  4. Informally solving by applying proportional reasoning
  5. Estimating the result of addition of decimal numbers
  6. Finding the area of a rectangle
Advice for Instruction | 1. Exploring problem-solving strategies | Deliver instruction

Deliver instruction (Lesson 1.2)

Lesson materials Lesson preview Lesson activities

Lesson materials

- *Lesson 1.2* “Collaboration and problem solving”
- Student Activity Book
- Large chart paper and markers (for teacher’s use)

Lesson preview

<table>
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<th>Activity</th>
<th>Goals</th>
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<tbody>
<tr>
<td>10 min</td>
<td>Opener</td>
<td>Think about what makes collaboration effective</td>
</tr>
<tr>
<td>30 min</td>
<td>Core activity</td>
<td>Apply effective collaboration strategies to solve a mathematical problem</td>
</tr>
<tr>
<td>20 min</td>
<td>Process homework</td>
<td>Learn about the routine that will be used to process homework</td>
</tr>
<tr>
<td>15 min</td>
<td>Consolidation activity</td>
<td>Learn about mathematical problem solving and reflect on how learning feels</td>
</tr>
<tr>
<td>5 min</td>
<td>Wrap up and introduce homework</td>
<td>Reflect on the day’s lesson and understand tonight’s homework assignment</td>
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Lesson activities

OPENER (10 minutes)

Students reflect on previous collaborative learning experiences and consider characteristics of successful partner or group work as well as difficulties encountered. [SAB, questions 1-2]

Online page 1

- In debriefing the opener, connect the collaborative work students did in the shape equation puzzles to the collaborative work students do in other areas, like science, social studies, or mathematics.
Online page 2

- Preview the activities and learning goals for the day’s lesson.

CORE ACTIVITY (30 minutes)

Students establish a list of class criteria for effective partner work and think about ways for overcoming partner-work challenges. Students work with their partners to solve a mathematics problem.

Online page 3

The class co-constructs criteria for effective partner work.

- Ask students how it helped to have a partner to work on the shape equations in Lesson 1.1. Inform students that solving many of the problems in the course will require that they work collaboratively, just as mathematicians and scientists often work together on complex problems. Point out that problem solving is not limited to only mathematics and science — all disciplines and careers require problem solving.
- Ask students for ideas of what makes group work successful, and what makes group work challenging. Here are some ideas that students might generate of behaviors that make group work successful: participation; encouragement; asking questions in your group before asking the teacher; listening to other students’ ideas; not criticizing each other.
- Transcribe the lists on two different charts: “Criteria for Quality Partner Work” and “Partner Work Challenges.” As you make the charts, inform students that they should also be recording the criteria for quality partner work in their activity books. [SAB, question 1]
- Once both lists have been shared, focus on the challenges with the following questions in mind:
  - What are some ways to address these challenges?
  - What have you tried in the past?
  - What are some things that have worked for you to overcome this particular challenge?
  - Are there things on our “Quality Partner Work” list that address these challenges?
  - What other items can we add to our “Quality Partner Work” list that will help solve the “Partner Work Challenges?”
- For each item on the challenges list, annotate its possible solution in another color, until all the items have a suggested solution that appears on the “Criteria for Quality Partner Work” chart.
- Let students know that the “Criteria for Quality Partner Work” chart will be posted in the classroom and that they will use it throughout the year to help guide their partner work.

Classroom strategy:
- Use the chart as an instructional tool throughout the upcoming weeks and months to help facilitate your students’ abilities to work collaboratively.
- As students are working with others, look for opportunities to recognize students for specific demonstrations of quality partner work.
- If a group is struggling, remind students of the “Criteria for Quality Partner Work” chart and ask the group members to look for solutions or actions that will help them get back on track.
- Some students may have had negative experiences in the past with working in groups. Watch for student pairs that require extra attention; help them foster good dispositions and work habits in working with others.
Online pages 4-5

Student pairs work collaboratively to solve a mathematical problem. Students should investigate and solve the Bike and Skateboard Problem using the tools they bring from middle school: trial and error, table-building, making sketches, etc.

A goal of the course is for students to experience the ways that mathematicians and scientists work together in the real world on complex problems. It is important to monitor students' progress and provide encouragement and feedback without trying to solve the problem for them. Students need opportunities to explore the data and tools as a team. Often, when students get frustrated they are not using their quality group work criteria. Point out to students that, as in real life, complex problem-solving situations require people to work harder and more effectively.

- Page 4: In this animation, students are given information about the Bike and Skateboard Problem. Students should identify key information in their activity books. [SAB, question 2]
- Page 5: Use panel 1 of the animation to check that students have identified the important information in the problem. Call on several students to share one fact each that they think is relevant to solving the problem. Then, have student pairs work together to solve the problem. [SAB, question 3-4] Encourage students to "think behind the shop door" to solve the problem. Allow student pairs about 15 minutes to work on this problem. If students are not sure how to get started, you can ask them questions to clarify their understanding of the problem as well as questions about their strategies for solving the problem:
  - Can you tell me what the problem states in your own words?
  - What’s the goal of the problem — what do you have to do?
  - Tell me how you’re thinking about trying to get an answer.
  - Would 10 skateboards and 11 bikes work? How do you know?

Push student pairs to write down their solution along with their solution strategy. Allow sufficient time for this activity so that each pair of students has a solution and strategy recorded in their booklet. Tell students that they will be discussing their solutions tomorrow, so their recorded notes and process will be important for jogging their memories. Use the second panel of the animation to allow students to check their solutions.
- Classroom strategy. As you monitor student progress with this task, pay careful attention to how students are handling mistakes, misunderstandings or dead ends. In the next lesson, you will encourage students to reflect on their problem-solving process with special emphasis on how they identified, learned from, and corrected mistakes. It will be helpful for you to take a few notes today as students are working so that you have some starting points for tomorrow’s discussion.

PROCESS HOMEWORK (20 minutes)

The routine for reviewing homework is introduced. Students use the routine to process the homework that was due today.

Online pages 6-9

Introduce the daily routine for processing homework.

- Tell students to take out the homework that was due today, Homework 1.1 and Staying Sharp 1.1. Refer them to the Homework-Processing Routine in the student booklet to discuss how homework will be reviewed each day. As you review the parts of the routine, model each step
to the class; be explicit about what students should be doing at each step. [SAB]

- Page 6: Review the steps that students should do with their partner as part of the homework processing routine.
- Page 7: Review the steps that will be completed as a class as part of the routine.
- Page 8: Review the steps that students should do on their own as part of the routine.
- Page 9: Review the steps that students should do as they finalize the processing of the homework.

Students use the routine to process the homework due today.

- Have students apply the Homework-Processing Routine to the homework assignments that were due today. As students review the homework together, circulate around the room. Gather data to determine if you need to have a brief follow-up to the partner conversations in order to clarify ideas or address misconceptions. As part of this process, you might ask one or two students to explain or present one of the homework problems.
- Try to keep the homework processing brief and to the point. You can share with the students that this routine must be conducted like a pit stop in a car race. Emphasize that every second counts.
- Classroom strategy. The stoplight symbols students place on their homework assignments will provide you with useful feedback about students’ understanding of key skills and concepts. Another purpose for having students use the stoplight symbols is to build a culture of self-regulation and responsibility among students for their own learning.

CONSOLIDATION ACTIVITY (15 minutes)

Students are introduced to a major theme for the unit, mathematical problem solving.

Online pages 10-12

- Page 10: Use this animation — comparing problem solving to exploring a maze — to prompt a brief discussion about the feelings described. Ask:
  - Did anyone experience any of these feelings when you were working on the shape equation puzzles or the Bike and Skateboard Problem?
  - Did you feel more confident that you could solve some of the problems and less confident when you got to some of the more challenging problems?
- Page 11: Play the animation to preview some questions students will explore this year in their algebra course.
- Page 12: Direct students to complete the questions in the consolidation activity in the Student Activity Book. These questions provide students with an opportunity to reflect on and write about problem solving. [SAB, questions 1-5]

WRAP UP AND INTRODUCE HOMEWORK (5 minutes)

Online page 13

Introduce tonight’s homework following the routine presented during the last class period:

- Homework 1.2
  Students continue their work with shape equation puzzles. Some of the equations in each puzzle involve multiplication that is not indicated through explicit symbols. Students also reflect on and write about group-work norms and the homework processing routine.
• **Staying Sharp 1.2**
  The main concepts and skills students will review in these problems are:
  1. Solving a system of two shape equations (with addition and multiplication)
  2. Continuing, describing, and extrapolating a pattern (represented with letters)
  3. Calculating a percentage from a verbal description
  4. Informally solving a two-step linear equation
  5. Subtracting of decimal numbers with regrouping
  6. Finding a missing dimension of a rectangle given its perimeter
Advice for Instruction | 1. Exploring problem-solving strategies | Deliver instruction

**Deliver instruction (Lesson 1.3)**

**Lesson materials**

- Lesson 1.3 “Mathematical presentations”
- Student Activity Book
- Large chart paper and markers (for use by student pairs)

**Lesson preview**

<table>
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<td>10 min</td>
<td>Opener</td>
<td>Think about what makes good mathematical communication</td>
</tr>
<tr>
<td>20 min</td>
<td>Core activity</td>
<td>Construct the criteria that will be used to evaluate presentations</td>
</tr>
<tr>
<td>10 min</td>
<td>Process homework</td>
<td>Learn from reviewing the homework due today</td>
</tr>
<tr>
<td>35 min</td>
<td>Consolidation activity</td>
<td>Discuss the importance of making and learning from mistakes; apply, share, and reflect upon strategies used to solve a mathematical problem</td>
</tr>
<tr>
<td>5 min</td>
<td>Wrap up and introduce homework</td>
<td>Reflect on the day’s lesson and understand tonight’s homework assignment</td>
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</table>

**Lesson activities**

**OPENER (10 minutes)**

Students think about what makes good communication in a mathematical presentation and work together with the class to list characteristics of good mathematical presentations in preparation for the presentation they will make to the class later today. They will consider the actions of speakers and audience members. Activities such as this one aim to create a sense of ownership for students. [SAB]

**Online page 1**

- In debriefing the opener, prepare a two-column class poster on chart paper to capture
students’ responses. Title the poster “Characteristics of Good Mathematical Communication,” with "Speakers" on one side and "Audience members" on the other side.

- Facilitate a brief whole-class discussion. Ask students to contribute criteria from their lists, and add them to the posters. Get class buy-in and participation by asking:
  - Do you think that’s one that should go on our poster?
  - How many other people had that on their list?
- Keep the poster visible in the room so that you can refer students to it in future weeks and months.

Online page 2

- Preview the activities and learning goals for the day’s lesson.

CORE ACTIVITY (20 minutes)

Online pages 3 and 4

Students learn about and discuss good communication skills, and review the presentation rubric.

- **Page 3**: Before playing the animation, clarify that the speakers are typically “givers” and audience members are “getters.” However, a good mathematics discussion should have all participants in both roles.
- Play the animation to provide additional, specific communication strategies for both givers and getters. Pause after panels 1 and add to the “Good Mathematical Communication” list as needed. With Panel 3, have students discuss the questions with a partner.
- **Classroom strategy.** If there is time, model active listening strategies by asking a volunteer to tell a brief story while you model appropriate body language and questions, such as “What I heard you say is... It sounds like you think....” Then share sample clarifying questions, such as “What approach should I use to get started? Can you restate the problems in another way? How does what we have already learned apply to this problem? What other examples could you share with me?”
- **Page 4**: Refer students to the chart listed in their activity books. [SAB, question 1] Tell students that this list for quality presentations includes characteristics for good mathematical communication, such as the ones they identified in the Opener. Tell students that they will receive a score on today’s presentation, and they can receive a top mark by meeting all of the criteria listed in the rubric.
- **Classroom strategy.** At this time, you may want to share a grading rubric with students. A sample is provided below which you can adapt to meet your needs. If you do choose to share it, take a minute to make sure everyone understands the term “rubric.”

<table>
<thead>
<tr>
<th>Speakers</th>
<th>4 3 2 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Clear write-up of your solution to the problem.</td>
<td></td>
</tr>
<tr>
<td>2. Clear, concise explanation as to why you believe your answer is correct.</td>
<td></td>
</tr>
<tr>
<td>3. Clear, concise explanation of solution strategy.</td>
<td></td>
</tr>
<tr>
<td>4. Both partners participate in presentation.</td>
<td></td>
</tr>
<tr>
<td>5. Both partners use strong, clear voices when making the presentation; both partners employ good posture and make eye contact.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Audience Members</th>
<th></th>
</tr>
</thead>
</table>

2 of 5
1. Give their full attention and respect to the presenter. 4 3 2 1
2. Take notes, as needed. 4 3 2 1
3. Are prepared to summarize the presenters’ mathematical argument or strategy. 4 3 2 1
4. Ask clarifying questions. 4 3 2 1
5. Are prepared to make connections between the presenters’ ideas and their own ideas. 4 3 2 1

4 = Meets criteria at a high level
3 = Meets criteria at a satisfactory / solid level
2 = Minimally meets criteria
1 = Does not meet criteria

Online page 5

Students prepare a class presentation to the Bike and Skateboard Problem.

- Distribute large chart paper and markers to each pair of students. Give students several minutes to prepare their presentations, allowing them time to complete their posters. [SAB, question 2] You will probably need to play the role of timekeeper during this process.
- Make explicit to students that they should strive to incorporate the criteria for effective presentations as they plan both their written and oral presentations. As they prepare their written presentation, they should also be thinking about what they will say when they make their oral presentation to the class. Oral presentations should address the following three categories:
  - The solution
  - A description of how they know their solution is correct
  - A description of the strategy used to solve the problem
- Remind students that the presentations should be brief. They do not have to share every step in their thought process as they solved the problem. The goal is to present a summary of the approach used and some of the thinking that was involved.
- Classroom strategy. You will not have time for every pair to present their solutions in the Consolidation activity, but all of their written presentations can be displayed. As you monitor and support student progress during this preparation time, look for variety in approaches and identify 3-5 pairs to share their oral presentations with the rest of the class. If possible, identify at least one or two pairs who are willing to discuss mistakes they made along the way.

PROCESS HOMEWORK (10 minutes)

Online page 6

Process the homework that was due today, Homework 1.2 and Staying Sharp 1.2.

CONSOLIDATION ACTIVITY (35 minutes)

Selected pairs of students present their solution and solution method to the Bike and Skateboard Problem.

Classroom strategy. All students should participate today as either audience members, speakers, or both. This will help create buy-in and accountability, and will set expectations for engagement.
This will also let students enact the criteria established by the class. Most important, each student will have a successful presentation experience in a low-stakes setting. Getting up in front of the class to speak can be intimidating. Most students will have success in solving the Bike and Skateboard Problem, and there are multiple ways to solve the problem. Therefore, this is a good problem to use for students’ first presentation experience. In making your selections, try to ensure a variety of solution methods will be modeled, and make an effort over time to balance the students who present across all members of the class.

Online pages 7 and 8

Students consider the role that mistakes will play in the course.

- Page 7: Have students individually reflect on the questions and then briefly discuss with a partner. [SAB, question 1]
- Page 8: Have students briefly discuss their responses to the prompts about the motto. [SAB, question 2] Facilitate a whole class discussion about students' responses to the prompts, highlighting specific places where mistakes are commonly expected, respected, and inspected.
- Acknowledge that math classes often focus strongly on correct answers, but that this class will also honor mistakes because inspecting and correcting them are so important to learning and the development of true understanding.

Online page 9

Students present their solutions to the Bike and Skateboard Problem.

- Inform students of the maximum amount of time that they will have to make their presentation. You may want to use a time limit of one minute. Have select pairs of students make their presentations to the class on the Bike and Skateboard Problem. As each pair presents, evaluate the presentation. Refer to the rubric above. You may want to adapt the rubric or devise your own evaluation system.
- Remind audience members of their criteria and encourage them to record notes or feedback as they listen and engage with the presenters. [SAB, questions 3 and 4]
- After each pair presents, use some of the following questions to hold audience members accountable:
  - Who can restate the strategy used by these students? (Check with the presenters to confirm.)
  - Who had a similar strategy?
  - Who has a clarifying question for these classmates?
  - How did these classmates learn from their mistakes or misunderstandings?
- Hold the presenters accountable using some of the following prompts:
  - Share something that was challenging about this task.
  - Were there misunderstandings or mistakes along the way?
  - What did you learn by identifying and correcting mistakes?
  - How is your strategy similar to the strategies of others?
- Once the selected pairs have presented, facilitate a brief whole-class conversation. Begin by focusing on the solution strategies:
  - Which presentations were similar?
  - How many different ways of solving the problems did we see?
  - What names could we give to some of the solution strategies?
• Then ask students about the presentations.
  ○ What were some of the things that you liked in the presentations?
  ○ Is there anything that we need to add to our poster of characteristics of good mathematical communication?

• Classroom strategy. Having students share their thinking around the work they do in the class is critical to building a “discourse community” or a “community of practice.” One hallmark of these types of communities is respectfulness for the contributions of others. You should work to build participation and respectfulness from the very first day of the course.

WRAP UP AND INTRODUCE HOMEWORK (5 minutes)

Online page 10

• Homework 1.3
  The mathematics needed to solve the first problem is similar to that needed for the Bike and Skateboard Problem; the context involves tickets sold (adult and children) for a movie. The second question asks students to reflect on the presentation that they gave in class today. The final question asks students to write a “mini-autobiography,” in which they share info to help you get to know students better as individuals and as math learners.

• Staying Sharp 1.3
  The main concepts and skills students will review in these problems are:
  1. Identifying the behavior of the additive identity in a shape equation
  2. Computing products and describing a pattern (related to place value)
  3. Reasoning about simple probability by varying the number of outcomes
  4. Informally solving by applying proportional reasoning
  5. Estimating the result of multiplication of decimal numbers
  6. Finding the surface area of a cube
Deliver instruction (Lesson 1.4)

Lesson materials

- Lesson 1.4 “Building a mathematical toolbox”
- Student Activity Book

Lesson preview

<table>
<thead>
<tr>
<th>Suggested Time</th>
<th>Activity</th>
<th>Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 min</td>
<td>Opener</td>
<td>Apply problem-solving strategies to solve a problem with multiple answers</td>
</tr>
<tr>
<td>30 min</td>
<td>Core activity</td>
<td>Analyze different problem-solving methods and apply problem-solving strategies to a mathematical problem</td>
</tr>
<tr>
<td>10 min</td>
<td>Process homework</td>
<td>Learn from reviewing the homework due today</td>
</tr>
<tr>
<td>25 min</td>
<td>Consolidation activity</td>
<td>Create a list of problem-solving strategies that can be used throughout the course</td>
</tr>
<tr>
<td>5 min</td>
<td>Wrap up and introduce homework</td>
<td>Reflect on the day’s lesson and understand tonight’s homework assignment</td>
</tr>
</tbody>
</table>

Lesson activities

OPENER (10 minutes)

Today’s opener sets the stage for one of the lesson’s core learning activities — an extension of the Bike and Skateboard Problem. The original Bike and Skateboard Problem had one solution that met the conditions of the problem. The Opener involves a problem with multiple solutions that meet the condition presented. [SAB, questions 1-2]

Online page 1

- In debriefing the opener, have students share all the different ways that 30¢ in change can be given, using quarters, dimes and nickels. Ask:
How many total ways did you find to give 30¢ in change?

- If students do not agree on an answer (there are 5 ways), you can ask students “How can we figure this out?” Have students contribute solutions. Write a list of the possibilities on the board as they do so. By making a list, you are modeling a problem-solving strategy. Ask students if there is a good way to organize the list. Use the phrase systematic list or organized list in the discussion.

Online page 2

- Preview for students the topic goals addressed in the day’s lesson and the lesson activities.

CORE ACTIVITY (30 minutes)

Students view two ways of solving the Bike and Skateboard Problem, and compare and contrast the two methods. Students then solve an extension of the Bike and Skateboard Problem — in which there are multiple solutions that meet the conditions outlined in the problem — and reflect on how the new problem is different from the original Bike and Skateboard Problem.

Online pages 3-6

Students analyze two strategies for solving the Bike and Skateboard Problem.

- Page 3: Show Lara’s solution strategy.
- Page 4: Give students time to complete the questions in their activity books. [SAB, questions 1a-1b] Facilitate a whole-class discussion with the reflection questions. Ask:
  - Did Lara’s method make sense? Do you have any questions about her method?
- Page 5: Show McKenna’s solution strategy.
- Page 6: Give students time to complete the questions in their activity books. [SAB, questions 2-3] Then facilitate a whole-class discussion using the reflection questions. Ask:
  - Did McKenna’s method make sense? Do you have any questions about her method?
  - Who thought Lara's solution made more sense? Why?
  - Who thought McKenna's solution made more sense? Why?

Online pages 7-9

Students work on an extension of the original Bike and Skateboard Problem and discuss their solution strategies.

- Page 7: Read through the new problem.
- Page 8: Give students time to work with their partners on the new problem. [SAB, question 4a-c] Circulate around the room as students work on the problem. Check progress and ask questions that help move students forward in their understanding of the problem as well as the strategy they will employ to solve the problem. Look for interesting solution strategies and for opportunities to highlight overcoming obstacles or learning from mistakes. You can call on these students to present their solution, solution strategies, and corrected mistakes during the debriefing of the activity.
- Classroom strategy. Tell students to keep track of their work, including taking notes about approaches that may not have helped solve the problem. Mathematicians sometimes try approaches that do not work, but these attempts can be a helpful learning experience. You can often build from unsuccessful attempts by using or modifying some part of the strategy.
- Invite a few student pairs to the class computer to enter their solutions in the animation and
discuss their solution strategies. The discussion of this activity should be brief and focused on a few key strategies.

- **Classroom strategy.** One way to hold students accountable for listening is to say,
  - *Raise your hand if you understand this strategy.*
  - *Leave your hand up if you can restate that strategy for those who are not clear yet.*

  Then have someone quickly restate the strategy. Or ask,
  - *How many of you used a similar strategy?*

- **Page 9:** Ask students to describe the key difference between the two Bike and Skateboard Problems they solved: one solution versus many solutions. Then have students answer the question in their activity books. **[SAB, question 4d]** Ask:
  - *Who feels confident that they’ve found all of the possible solutions?*
  - *How can you be sure that you found all of the solutions?*
  - *How would you convince someone else that you’ve found all of the solutions?*

  Be sure to highlight any different problem-solving strategies (pictures, tables, systematic lists) that students use in their justifications.

**PROCESS HOMEWORK (10 minutes)**

**Online page 10**

- Process the homework that was due today, *Homework 1.3* and *Staying Sharp 1.3.*

**CONSOLIDATION ACTIVITY (25 minutes)**

Students reflect on the strategies they used for solving problems during the opening days of the course.

**Online page 11**

- Give students time to reflect on and write responses to the two questions that are posed. **[SAB, questions 1-2]** Then facilitate a whole-class discussion.
- Briefly discuss the idea of a problem-solving toolbox that is presented.

**Online pages 12-13**

Students work with their partners and then with the class to generate a list of problem-solving strategies.

- **Page 12:** Give students a few minutes to work with their partner to generate a list of problem-solving strategies. **[SAB, question 3]**
- **Page 13:** Create a class poster titled “Problem-Solving Strategies Toolbox.” This poster will be posted in the classroom and the class will add to it in future lessons. Ask students to contribute strategies. While honoring student contributions, you should guide the discussion and make decisions about what goes on the poster. Here are questions to ask to Involve the class in making decisions about what goes on the poster:
  - *Is that strategy similar to one that we already have on the poster?*
  - *What do you think? Does that sound like a strategy that we use when we solve problems?*
  - *Should we add that one to our list?*
- Direct students to write down the information from the poster in their activity book. **[SAB, question 4]** Even though students have just brainstormed a list of problem-solving strategies,
it is important for students to have a more polished list, and one that was created through a class discussion process. Students can use this list for future reference as they continue in the course.

Online page 14

Students reflect on the learning in this topic.

- Give students several minutes to think and write about the focus questions. [SAB, question 5]. Debrief this portion of the activity by allowing students to share a few of their responses to the first and last questions. These questions will allow students to share their insights on this topic with their classmates.

WRAP UP AND INTRODUCE HOMEWORK (5 minutes)

Online page 15

- Homework 1.4
  The mathematics needed to solve the first problem is similar to that needed for the Bike and Skateboard Problem; wagons and tricycles are used instead of bicycles and skateboards. The second and fifth questions ask students to reflect on and write about some things related to their mathematics learning in the first several days of the course.

- Staying Sharp 1.4
  The main concepts and skills students will review in these problems are:
  1. Solving a system of two shape equations (with addition and like terms)
  2. Computing sums of digits (in multiples of 9) and describing a pattern
  3. Calculating a percentage from a verbal description
  4. Informally solving a multi-step linear equation
  5. Dividing decimal numbers
  6. Finding the perimeter of a rectangle with fractional side lengths
Deliver instruction (Lesson 1.5)

Lesson materials

- Lesson 1.5 “Problem solving with patterns”
- Student Activity Book
- Algebra tiles

Lesson preview

<table>
<thead>
<tr>
<th>Suggested Time</th>
<th>Activity</th>
<th>Goals</th>
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</thead>
<tbody>
<tr>
<td>10 min</td>
<td>Opener</td>
<td>Investigate the idea of absolute value by looking at opposite numbers, numbers that lie on the opposite sides of zero</td>
</tr>
<tr>
<td>15 min</td>
<td>Core activity</td>
<td>Apply algebraic reasoning and number patterns to solve problems</td>
</tr>
<tr>
<td>10 min</td>
<td>Process homework</td>
<td>Learn from the homework due today</td>
</tr>
<tr>
<td>40 min</td>
<td>Consolidation activity</td>
<td>Learn about automaticity and apply this concept to adding and subtracting signed numbers</td>
</tr>
<tr>
<td>5 min</td>
<td>Wrap up and introduce homework</td>
<td>Reflect on the day’s lesson and understand tonight’s homework assignment</td>
</tr>
</tbody>
</table>

Lesson activities

OPENER (10 minutes)

Students investigate the concept of absolute value by looking at numbers that lie on opposite sides of zero on the number line.

Online pages 1-2

- Page 1: Students work on the Opener. [SAB, questions 1-2]
- Page 2: Review the terms on this page with students in debriefing the Opener. Students will build on the concept of opposite numbers later in this lesson when they work on adding and subtracting signed numbers. Students will also build on the concept of zero pairs later in this
lesson when they learn the formal definition of additive inverses.

Online page 3

- Preview the activities and learning goals for the day’s lesson.

CORE ACTIVITY (15 minutes)

Students develop their numerical fluency by studying square box problems.

Online pages 4-5

Students are introduced to square box problems, and recognize and use patterns.

- Page 4: Introduce the square box problem type and have students study the example. Ask students to look for the relationships between the left and right parts of the box and the number at the top of the box and then the number at the bottom of the box. [SAB, questions 1-2]
- Page 5: Students complete six square box problems on their own and then review their answers with the class. Remind students to use the pattern that they just discovered. Note that the final problems involve negative numbers. [SAB, questions 3-4]

PROCESS HOMEWORK (10 minutes)

Online page 6

- Process the homework that was due today, Homework 1.4 and Staying Sharp 1.4.

CONSOLIDATION ACTIVITY (40 minutes)

Students are introduced to the “learning to learn” concept of “automaticity.” They begin their work with the unit’s focus skill of operations with signed numbers as they review adding and subtracting signed numbers as a skill for which automaticity is needed. Concrete models and patterning are used in the work with signed numbers.

Online pages 7-8

Examples from sports and music are used to introduce the idea of automaticity, and then connect that idea to learning mathematics.

- Read through the introductory material on automaticity as a class. Engage students by asking them to discuss areas in their own lives where “becoming automatic” is important. Then give students time to complete the reflection questions in their activity books. [SAB, questions 1-2]
- Classroom strategy. Consider calling on different students to read aloud one paragraph each. Or you might choose to use a “paired reading” format, in which each pair of students takes turns reading aloud a part of the passage to his/her partner. An advantage of a paired reading protocol is that every student in the room can have an opportunity to read.

Online pages 9-10

Students consider the need to build proficiency and skill with signed numbers in connection to the
theme of automaticity. Students are also introduced to the notion of a “conceptual model” and the need to build conceptual understanding in addition to automaticity.

- Page 9: Discuss how adding and subtracting signed numbers is a skill for which students need automaticity in algebra. Use the animation to review what students already know about signed numbers.
- Page 10: Discuss with students the meaning of “conceptual model.” Explain that having conceptual understanding and automaticity together will give them a more powerful understanding of mathematics.

**Online pages 11-14**

Students use algebra tiles and number line activities to help build understanding of the underlying mathematics of signed numbers.

**Classroom strategy.** As you are facilitating this portion of the lesson, look for opportunities to reinforce the class motto of “Mistakes are expected, inspected, and respected.” Encourage students to share how they are overcoming some of their misunderstandings about integers through the use of the models and the common mistakes that are made along the way.

- Page 11: Put the students in small groups and distribute one set of algebra tiles to each student. Have students sort the tiles by size and color. Ask students to focus on only the "positive" tiles. Explain that the dimensions of the positive unit tile are 1 by 1. Use these questions to build student understanding of the tile models:
  - What are the dimensions of one of the long, skinny tiles? How do you know? (1 by \(x\) - one unit tile on the end, an "unknown" number of unit tiles on the long side)
  - What is the area of one of the long, skinny tiles? (\(x\) square units)
  - What are the dimensions of one of the large, square tiles? How do you know? (\(x\) by \(x\) - each side of the square is exactly as long as the longest side of one of the \(x\) tiles)
  - What is the area of one of the large square tiles? (\(x^2\) square units)

  Once students have developed their understanding of the tiles, they should record this information in their activity books. [SAB, questions 3-4]

- Page 12: Define “additive inverses” and “zero pairs.” Use panel 1 of the animation to model the addition of an integer and its opposite as the students follow along with their own tiles. [SAB, question 5a] Ask:
  - Why did we get zero for an answer?
  - What other addition problems can you think of that will have 0 for an answer?

  Then have students use their tiles to model the next two addition problems in the animation. Invite several different students to come to the projection device and demonstrate their models. [SAB, questions 5b and 5c] Use panels 2 and 3 of the animation as needed to support students’ demonstrations.

- Show panel 4 of the animation to introduce how to apply the definition of subtraction to rewrite an integer subtraction problem as an addition problem. Have students use their tiles to model the equivalent addition problem and record their work [SAB, question 5d]. Invite a student to the projection device to demonstrate his or her model. Use panel 4 as needed to support the demonstration.

- **Classroom strategy.** Algebra tiles illustrate the process of working with integers, equations, and area models in a concrete and meaningful way for all learners. The key is to move the students beyond the concrete model into a facility with integer arithmetic that does not require the model.
Page 13: Demonstrate the number line addition model using panels 1 through 3 of the animation, then ask students to try the next example on their own. [SAB, question 6] Use panel 4 to hint at the connection between subtraction and addition, but solicit answers from students before using panel 5 to reveal the model.

Page 14: Have students work with their partners to answer questions 7 and 8 in their activity books. [SAB, questions 7-8] Then ask students to generalize the patterns of integer addition based on their work in the lesson. Some summarizing questions to ask:

- How can the patterns modeled by the algebra tile and number line examples help you solve problems without building models?
- How can these patterns be extended to use to solve problems involving fractions and decimals?

Online page 15

Students work on a math journal to capture their understanding of the vocabulary terms introduced in this activity.

Math journals will be used in this course to help students organize understandings related to important mathematical concepts, processes, and vocabulary. The journals will consist of either two columns or three columns and thus will sometimes be referred to as “double-column” or “triple-column journals.” By utilizing a straightforward format, the journals help students stay focused on the key ideas. In addition to helping students organize key information, the journals can be a reference for students as they work on future activities and prepare for tests.

- Discuss the purpose of math journals and then allow students about 7-8 minutes to complete this math journal. [SAB, question 9]
- Classroom strategy. Since this is the first math journal that students will be completing, it is important that students have several good models of a journal entry. With this in mind, display a journal entry that you have written, and then do a Think Aloud modeling the types of suggestions you would recommend for its improvement. Then ask students to volunteer to read their entries and have the class offer suggestions for ways in which the entries could be improved.

WRAP UP AND INTRODUCE HOMEWORK (5 minutes)

Online page 16

- Homework 1.5
  Students practice working with simple absolute value and signed number problems, including applying the concrete models of algebra tiles and the number line to their work with operations on signed numbers.
- Staying Sharp 1.5
  The main concepts and skills students will review in these problems are:
  1. Solving a system of three shape equations (by identifying the multiplicative identity)
  2. Computing the sum and product of two given integers
  3. Applying proportional reasoning to solve a contextual problem
  4. Informally solving a system of two conditions in a geometric context
  5. Rounding decimal numbers
  6. Finding the volume of a rectangular solid